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BARYON DENSITY AND THE DILATED CHIRAL QUARK MODEL

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Abstract

We calculate perturbatively the effect of density on hadronic properties using the chiral quark model implemented by the QCD trace

anomaly to see the possibility of constructing Lorentz invariant Lagrangian at finite density. We calculate the density dependent masses of the constituent quark, the scalar field and the pion in one-loop order using the technique of thermo field dynamics. In the chiral limit, the pion remains massless at finite density. It is found that the tadpole type corrections lead to the decreasing masses with increasing baryon density, while the radiative corrections induce Lorentz-symmetry-breaking terms. We found in the large N_c limit with large scalar mass that the tadpoles dominate and the mean-field approximation is reliable, giving rise a Lorentz-invariant Lagrangian with masses decreasing as the baryon density increases.

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1 *Introduction*

It has been discussed by a number of authors that QCD trace anomaly plays an important role at finite density and temperature. Dilaton fields which mimic the QCD trace anomaly are used to scale non-linear σ-model to study phase transitions[1][2] and Nambu-Jona-Lasinio model to study quark and gluon condensate[3] at finite density. A relativistic hadronic model with dilaton fields for nuclear matter and finite nuclei is constructed and applied to the one-baryon loop level to finite nuclei[4]. It has been also suggested by Brown and Rho [2] that the scale anomaly of QCD suitably incorporated into an effective chiral Lagrangian allows one to study dense-medium effects in a simple mean-field approximation provided the parameters of the Lagrangian scaled in medium. The predicted scaling law (called "BR scaling") has been found to be consistent with a variety of nuclear phenomena [5, 6, 7]. Starting with chiral quark model of constituent quarks and pions that incorporates the QCD scale anomaly, Beane and van Kolck[8] showed that the "mended symmetry" of Weinberg [9] can be realized in the dilaton limit with the effective Lagrangian. As discussed in [5], there is a close relation between the physics of BR scaling and that of the dilated chiral quark model of Beane

and van Kolck [8]. Thus the dilated chiral quark model is supposed to describe in some qualitative and heuristic way the state of matter near the chiral phase transition much as BR scaling is to describe the "dropping" hadron masses near the phase transition [10]. In the recent work[11], this aspect of the dilated chiral quark model was exploited to predict the temperature dependence of hadronic properties near the phase transition: this calculation confirms what has always been tacitly assumed, namely, a more rapid decreasing of f_π, m_σ, and m with temperature in the presence of matter density compared with the zero-density σ model. In these developments, the question as to how relevant the dilaton limit is to dense nuclear matter was not, however, properly addressed: for example, how the Lorentz-invariant description of dilaton limit in the presence of Fermi sea could be a good approximation.

In this paper, we make an attempt to answer the question by calculating density effects starting from the chiral Lagrangian implemented with the QCD trace anomaly. We calculate perturbatively the effect of density on hadronic properties using the chiral quark model with dilaton fields from which Bean and van Kolck derive, in the dilaton limit, the dilated chiral quark model. Since we are interested mostly in density effects, we take the

low-temperature limit and focus on the parts that explicitly depend on the density. Using large N_c approximation[12], we show that the terms which break Lorentz invariance can be neglected at large N_c and for large dilaton mass and discuss the physical relevance of Lorenz-invariant Lagrangian at finite density.

The paper is structured as follows. In section 2, we present the chiral quark model with the dilaton fields and analyze the one-loop structures of the model using large N_c approximation. We calculate the density-dependent masses of the pion (m_π), the constituent quark (m) and the scalar field (m_χ) in section 3 at one-loop order using the technique of thermo field dynamics. In section 4, we discuss the relevance of mean-field approximation in our perturbative calculations and summarize the results. The basics of the thermo field dynamics are summarized in Appendix I. Some details of our calculations are given in Appendix II.

2 *Effective Chiral Lagrangian*

The effective chiral-quark Lagrangian[13] implemented with the QCD

conformal anomaly is[8]

$$L = \bar{\psi} i(\partial\!\!\!/ + V\!\!\!/)\psi + g_A \bar{\psi}\ A\!\!\!/ \gamma_5 \psi - \frac{m}{f_d}\bar{\psi}\psi\chi + \frac{1}{4}\frac{f_\pi^2}{f_d^2} tr(\partial_\mu U \partial^\mu U^\dagger)\chi^2$$
$$+\frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(\chi) + ... \qquad (1)$$

where $V_\mu = \frac{1}{2}(\xi^\dagger\partial_\mu\xi + \xi\partial_\mu\xi^\dagger)$, $A_\mu = \frac{1}{2}i(\xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger)$ with $\xi^2 = U = \exp(\frac{i2\pi_i T_i}{f_\pi})$ and $g_A = 0.752$ is the axial-vector coupling constant for the constituent quark [1]. The potential term for the dilaton fields, [2]

$$V(\chi) = -\frac{m_\chi^2}{8f_d^2}[\frac{1}{2}\chi^4 - \chi^4 \ln(\frac{\chi^2}{f_d^2})], \qquad (2)$$

reproduces the trace anomaly of QCD at the effective Lagrangian level. We assume that this potential chooses the "vacuum" of the broken chiral and scale symmetries, $< 0|\chi|0 >= f_d$ and the dilaton mass is determined by $m_\chi^2 = \frac{\partial^2 V(\chi)}{\partial\chi^2}|_{\chi=f_d}$.

After shifting the field, $\chi \to f_d + \chi'$, eq.(1) becomes

$$L = \bar{\psi} i(\partial\!\!\!/ + V\!\!\!/)\psi + g_A \bar{\psi}\ A\!\!\!/ \gamma_5 \psi - m\bar{\psi}\psi - \frac{m}{f_d}\bar{\psi}\psi\chi'$$

[1] Since the role of gluons in the chiral quark model is negligible, we will ignore the terms containing gluons[13] [14].

[2] The scalar field that figures in the dilaton limit must be the quarkonium component that enters in the trace anomaly, not the gluonium component that remains "stiff" against the chiral phase transition. See [5] for a discussion on this point.

$$
\begin{aligned}
&+ \quad \frac{1}{4} f_\pi^2 tr(\partial_\mu U \partial^\mu U^\dagger) + \frac{1}{2}\frac{f_\pi^2}{f_d} tr(\partial_\mu U \partial^\mu U^\dagger)\chi' + \frac{1}{4}\frac{f_\pi^2}{f_d^2} tr(\partial_\mu U \partial^\mu U^\dagger)\chi'^2 \\
&+ \quad \frac{1}{2}\partial_\mu \chi' \partial^\mu \chi' - V(f_d + \chi') + \dots \, . \qquad (3)
\end{aligned}
$$

Expanding U in terms of pion fields and collecting the terms relevant for one-loop corrections, the Lagrangian for the fluctuating field can be written as

$$
\begin{aligned}
L \quad &= \quad \bar{\psi} i \not{\partial} \psi - m\bar{\psi}\psi - \frac{m}{f_d}\bar{\psi}\psi\chi' - \frac{g_A}{f_\pi}\bar{\psi} \not{\partial}\pi\gamma_5\psi - \frac{1}{2f_\pi^2}\epsilon_{abc}T_c\bar{\psi} \not{\partial}\pi^a\pi^b\psi \\
&+ \quad \frac{1}{2}\partial_\mu\pi\partial^\mu\pi + \frac{f_\pi^2}{f_d}\partial_\mu\pi\partial^\mu\pi\chi' + \frac{1}{2f_d^2}\partial_\mu\pi\partial^\mu\pi\chi'^2 \\
&+ \quad \frac{1}{2}\partial_\mu\chi'\partial^\mu\chi' - \frac{1}{2}m_\chi^2\chi'^2 - \frac{5}{6}\frac{m_\chi^2}{f_d^2}\chi'^3 + \dots \, . \qquad (4)
\end{aligned}
$$

We analyze the Lagrangian using the large N_c approximation at one-loop order. From 't Hooft and Witten[12], we know the N_c dependence of the properties of mesons and baryons. Some of them, which are relevant to us, are summarized as follows: Three meson vertiex is of order $\frac{1}{\sqrt{N_c}}$. Baryon-meson-baryon vertex is of order $\sqrt{N_c}$. But, since the constituent quarks in eq.(1) have color indices, there should be some modification to the meson-fermion vertex. The mesons are created and annihilated by quark bilinears,

$$
B = N_c^{-\frac{1}{2}}\bar{\psi}^a\psi^a \qquad (5)
$$

If we consider matrix element of eq.(5) between two color-singlet baryons,

the order is $N_c/N_c^{\frac{1}{2}}$ since N_c quarks are involved to be annihilated by B. However in our case the ground-states are the constituent quarks with definite color and only one quark which has same color with ground-state quarks can contribute the matrix element; thus the quark-meson-quark vetrex is of order $1/N_c^{\frac{1}{2}}$. So quark-meson-quark vertex is of order $\frac{1}{\sqrt{N_c}}$ while there are no changes in N_c counting for mesonic vertices. Now we can determine N_c dependences of one-loop diagrams. The tadpole contribution to consitutent quark mass, Fig.2, is of order $N_c(\frac{1}{\sqrt{N_c}})^2 = 1$ and that from Fig.3 is of order $(\frac{1}{\sqrt{N_c}})^2 = \frac{1}{N_c}$. The two diagrams, Fig. 4 and Fig. 5, for dilaton mass corrections are of the same N_c order.

3 *Perturbative calculations*

We compute the diagrams representing mass corrections at one-loop approximation. At very low temperature, the Bose-Einstein distribution function $n_B(k) = \frac{1}{e^{\beta k}-1}$ goes to zero but the Fermi-Dirac distribution function $n_F(p) = \frac{1}{e^{\beta(p-\mu)}+1}$ becomes $\theta(\mu - p)$. Thus in our case, the finite-temperature and -density corrections come from quark propagators. In our calculations of

Feynman integrals, we shall follow the method of Niemi and Semenoff [15]. In the case of renormalization of finite-temperature QED [16] one encounters a new singularity like $\frac{1}{k}\frac{1}{e^{\beta k}-1}$. However in our case, because pion-quark vertices depend on momentum, we do not have any additional infrared singularities at finite temperature and density. At finite temperature and density, the lack of explicit Lorentz invariance causes some ambiguities in defining renormalized masses. The standard practice is to define density-dependent (or T-dependent) mass corrections as the energy of the particle at $\vec{p} = 0$ [17]. In our case, this definition may not be the most suitable one because of the Fermi blocking from the fermions inside the Fermi sphere. We find it simplest and most convenient to define the mass at $\vec{p} = 0$.

Pion mass

We first consider pion mass in baryonic matter at low temperature. There are three density-dependent diagrams, Fig. 1, that may contribute to pion mass corrections. The first one, Fig. 1a, vanishes identically due to isospin symmetry. Explicit calculation of the two diagrams in Fig. 1b gives, for

$\beta \to \infty$,

$$
\begin{aligned}
\Sigma_\pi(p^2) &= i(\frac{g_A}{f_\pi})^2 tr \int \frac{d^4k}{(2\pi)^4} \not p \gamma_5 T^a (\not p + \not k + m) \cdot \not p \gamma_5 T^a (\not k + m) \\
& [\frac{i}{(p+k)^2 - m^2} (-2\pi) \delta(k^2 - m^2) \sin^2 \theta_{k_0} \\
& + \frac{i}{k^2 - m^2} (-2\pi) \delta((k+p)^2 - m^2) \sin^2 \theta_{k_0 + p_0}]. \qquad (6)
\end{aligned}
$$

With the change of variable on the second term, $p+k \to k$, the above integral can be rewritten as

$$
\begin{aligned}
\Sigma_\pi(p^2) &= -2(\frac{g_A}{f_\pi})^2 \int \frac{d^4k}{(2\pi)^3} [\frac{-p \cdot k(p^2 + 2k \cdot p) + 2m^2 p^2}{p^2 + 2k \cdot p} \\
& \quad + \frac{p \cdot k(p^2 - 2k \cdot p) + 2m^2 p^2}{p^2 - 2k \cdot p}] \delta(k^2 - m^2) \sin^2 \theta_{k_0} \\
&= -2p^2 (\frac{g_A}{f_\pi})^2 \int \frac{d^4k}{(2\pi)^3} [\frac{2m^2}{p^2 + 2k \cdot p} + \frac{2m^2}{p^2 - 2k \cdot p}] \\
& \quad \times \delta(k^2 - m^2) \sin^2 \theta_{k_0}. \qquad (7)
\end{aligned}
$$

Since the pion self-energy is proportional to p^2, the pole of the pion propagator does not change. Hence the pion remains massless as long as there is no explicit chiral symmetry breaking. Naively we might expect that the pion may acquire a dynamical mass due to dynamical screening from thermal particles or particle density, which is the case for QED [18] even with gauge invariance. The reason for this is the derivative pion coupling to the quark field. In eq.(7), the terms with $p \cdot k$ may give rise to terms which are not

proportional to p^2, which lead to the pion mass correction after dk integration. However, those terms are proportional to $p^2 + 2k \cdot p$ or $p^2 - 2k \cdot p$ which are cancelled by the their denominators and contribute nothing after the dk integration. This feature has been explicitly demonstrated in eq.(7).

Since the pole of the pion propagator does not change, we can take the renormalization point at $p^2 = 0$. Then the integrand itself vanishes, so there is no wave-function renormalization for the pion in dense medium.

Quark mass

At one-loop order, the quark self-energy is given by the diagrams of Fig. 2 and Fig. 3. The diagram Fig. 2 gives

$$\Sigma_Q^{(1)}(p) = i(-i\frac{m}{f_d})^2 \frac{i}{-m_\chi^2} \rho_s \tag{8}$$

where ρ_s is the scalar density obtained from the fermionic loop with thermal propagator,

$$\begin{aligned} \rho_s &= -tr \int \frac{d^4p}{(2\pi)^4} (-2\pi)(\not{p} + m)\delta(p^2 - m^2) sin^2\theta_{p_0} \\ &= \frac{m}{\pi^2}\theta(\mu - m)[\mu\sqrt{\mu^2 - m^2} - m^2 ln(\frac{\mu + \sqrt{\mu^2 - m^2}}{m})] \end{aligned} \tag{9}$$

where $\vec{p}_F^2 = \mu^2 - m^2$. With I defined as $\rho_s = 4mI$, we get

$$\Sigma_Q^{(1)}(p) = -(\frac{m}{f_d})^2 \frac{4m}{m_\chi^2} I \tag{10}$$

The radiative correction from the pion field in Fig. 3a is

$$\Sigma_Q^{3a}(p) = \frac{3}{8}(\frac{g_A}{f_\pi})^2[(\not{p} + m)I]. \tag{11}$$

Finally, the radiative correction due to the χ-field in Fig. 3b is found to be

$$\Sigma_Q^{3b}(p) = -(\frac{m}{f_d})^2[\not{J} - \frac{m}{2m_\chi^2} I] \tag{12}$$

where $\not{J}$ is defined

$$\not{J} \equiv \int \frac{d^4k}{(2\pi)^3} \frac{\not{k}}{2m^2 - 2p \cdot k - m_\chi^2} \delta(k^2 - m^2) \sin^2 \theta_{k_0}. \tag{13}$$

In sum, the self-energy of the quark with four momentum $(E, \vec{p})$ can be written in the form

$$\Sigma_Q(E, \vec{p}) = aE\gamma_0 + b\vec{\gamma} \cdot \vec{p} + c - (\frac{m}{f_d})^2 \frac{4m}{m_\chi^2} I \tag{14}$$

where a, b, and c are

$$\begin{aligned} a &= \frac{3}{8}(\frac{g_A}{f_\pi})^2 I - \frac{1}{E}(\frac{m}{f_d})^2 J^0 \\ b &= -\frac{3}{8}(\frac{g_A}{f_\pi})^2 I + (\frac{m}{f_d})^2 \frac{1}{\vec{p}^2} \vec{J} \cdot \vec{p} \\ c &= \frac{3}{8}(\frac{g_A}{f_\pi})^2 mI + (\frac{m}{f_d})^2 \frac{m}{2m_\chi^2} I \end{aligned} \tag{15}$$

where we have used the relation [16], $\vec{J}\cdot\vec{\gamma} = \frac{\vec{J}\cdot\vec{p}\ \vec{p}\cdot\vec{\gamma}}{\vec{p}^2}$. The detailed calculations of J^0 and $\vec{J}\cdot\vec{p}$ are given in appendix II.

From eq.(15), we can see that the scalar field radiative corrections violate the Lorentz symmetry, that is, $a \neq -b$, unless $\frac{1}{E}J^0 = \frac{1}{\vec{p}^2}\vec{J}\cdot\vec{p}$. On the other hand, the pion radiative corrections in a and b preserve Lorentz covariance. Since we have only $O(3)$ symmetry in the medium for the quark propagation, we will adopt the conventional definition of mass as a zero of the inverse propagator with zero momentum. The inverse propagator of the quark is given by

$$\begin{aligned} G^{-1}(E,p) &= \not{p} - m - \Sigma(E,\vec{p})) && (16) \\ &= E(1-a)\gamma_0 - (1+b)\vec{\gamma}\cdot\vec{p} - c - m && (17) \end{aligned}$$

The mass is now defined as the energy which satisfies $det(G^{-1}) = 0$ with $\vec{p}^2 = 0$,

$$(1-a)E = c + m. \tag{18}$$

Since a and c are perturbative corrections, eq.(18) can be approximated to the leading order as

$$E = m - (\frac{m}{f_d})^2 \frac{4m}{m_\chi^2} I + c + am. \tag{19}$$

If we define the mass as the energy of the particle at finite density, the quark mass becomes

$$
\begin{aligned}
m^* &= m - (\frac{m}{f_d})^2 \frac{4m}{m_\chi^2} I \\
&\quad + \frac{3}{4}(\frac{g_A}{f_\pi})^2 m I \\
&\quad - (\frac{m}{f_d})^2 J^0 + (\frac{m}{f_d})^2 \frac{m}{2m_\chi^2} I. \qquad (20)
\end{aligned}
$$

The first line in eq.(20) is just the result of the mean-field approximation, in which only the tadpole diagram, Fig. 2, is taken into account, as further elaborated on in the section 4. While the dropping of the quark mass with density is obvious in the mean-field approximation, it is not clear whether it is still true when the radiative corrections, Fig.3, are included as in the second and the third lines in eq.(20). Thus the result obtained at the one-loop order does not indicate in an unambiguous way that the quark mass is scaling in medium according to BR scaling [2]. The specific behavior depends on the strength of the coupling constants involved in the theory, g_A, m, f_π and f_d. This does not seem to be the correct physics for BR scaling as evidenced in Nature. However, as discussed in section 2, the tadpole contribution to consitutent quark mass, Fig.2, is of order 1 and that from Fig.3 is of order $\frac{1}{N_c}$. So we can neglect the Fig.3 for large N_c and obtain in-medium quark

mass,

$$m^* = m - (\frac{m}{f_d})^2 \frac{4m}{m_\chi^2} I. \tag{21}$$

In the large N_c limit, therefore, the quark propagator retains Lorentz covariance and the mass does decrease according to the mean-field approximation.

Scalar mass

Now consider the mass shift of the dilaton field (see Fig. 4 and Fig. 5). The tadpole diagram, Fig. 4, gives

$$\begin{aligned}\Sigma_\chi^{(2)}(p) &= -\frac{5}{6}\frac{m_\chi^2}{f_d}(-i\frac{m}{f_d})\rho_s(\frac{i}{-m_\chi^2})3! \\ &= -20(\frac{m}{f_d})^2 I \end{aligned} \tag{22}$$

where the factor 3! comes from the topology of the diagram. This corresponds to the mean-field approximation at one-loop order.

The analytic expression for the contribution from Fig. 5 can be obtained in the large scalar mass approximation, $m^2 << m_\chi^2$,

$$\begin{aligned}\Sigma_\chi^{(5)}(p) &= \frac{m^2}{m_\chi^2}\frac{8}{\pi^2}(\frac{m}{f_d})^2[\frac{1}{2}\theta(\mu - m)(\mu\sqrt{\mu^2 - m^2} - m^2\ln(\frac{\mu + \sqrt{\mu^2 - m^2}}{m})) \\ &\quad - \frac{E^2}{m_\chi^2}(\frac{\mu(\mu^2 - m^2)^{3/2}}{4m^2} + \frac{\mu\sqrt{\mu^2 - m^2}}{8}\end{aligned}$$

$$
- \frac{m^2}{8} \ln(\frac{\mu + \sqrt{\mu^2 - m^2}}{m}))]. \tag{23}
$$

Note that the terms on the second and the third lines contribute only to the *energy* of the dilaton field and hence break Lorentz invariance. This contribution, (23), however can be neglected since it is suppressed compaired to that of tadpole, eq.(22), by $\frac{m^2}{m_\chi^2}$. Hence the Lorentz invariance is maintained approximately in the large scalar mass approximation. Now the renormalized dilaton mass becomes

$$
m_\chi^{*2} = m_\chi^2 - 20(\frac{m}{f_d})^2 I. \tag{24}
$$

The mass of the dilaton drops as density increases. This is consistent with the tendency for the scalar to become dilatonic at large density, providing an answer to the question posed at the beginning.

4 *Discussion*

So far, we have not taken into account the fact that introducing the thermal fluctuation can cause further shifting of the vacuum expectation value of the dilaton field. The vacuum expectation value of χ'-field in eq.(3) is zero, $< 0|\chi'|0 >=0$, at zero density. The introduction of the new ground state,

$|F>$, which are already occupied by constituent quarks with $E<E_f$(fermi energy), implies the necessity of shifting the vacuum. Imposing the condition that all tadpole graphs vanish on the physical vacuum, in terms of Weinberg's notation[19], the condition is

$$(\frac{\partial P(\chi')}{\partial \chi'})_{\chi'=<F|\chi'|F>} + \tilde{T} = 0 \tag{25}$$

where $P(\chi')$ is a polynomial in χ' and $\tilde{T}$ is the sum of all tadpole graphs. In our case, we are interested in the small change of vacuum expectation value, $\frac{<F|\chi'|F>}{f_d} < 1$. So we can drop higher powers of the χ'-field and retain only the tadpoles that depend on density at one-loop order. Then the eq.(25) reads

$$-m_\chi^2 \chi_0 - \frac{m}{f_d}\rho_s = 0 \tag{26}$$

where $\chi_0 \equiv < F|\chi'|F >$. Then, we get the vacuum expectation value of the χ'-field.

$$\chi_0 = -\frac{m}{m_\chi^2 f_d}\rho_s \tag{27}$$

This is equivalent to Fig. 1 without the external quark line. With the shift of the χ'-field around χ_0, i.e. $\chi' \to \chi_0 + \tilde{\chi}$ in eq.(3) or $\chi \to f_d + \chi_0 + \tilde{\chi}$ in eq.(1), the Lagrangian is modified effectively to

$$L \;=\; \bar{\psi} i(\partial\!\!\!/ + V\!\!\!/)\psi + g_A \bar{\psi} A\!\!\!/ \gamma_5 \psi - \frac{m}{f_d}(f_d + \chi_0)\bar{\psi}\psi - \frac{m}{f_d}\bar{\psi}\psi\tilde{\chi}$$

$$
\begin{aligned}
&+\frac{1}{4}\frac{f_\pi^2}{f_d^2}(f_d+\chi_0)^2 tr(\partial_\mu U\partial^\mu U^\dagger)\\
&+\frac{1}{2}\partial_\mu\tilde{\chi}\partial^\mu\tilde{\chi}-V(f_d+\chi_0+\tilde{\chi})+\cdots
\end{aligned}
\tag{28}
$$

where the ellipsis stands for the interaction terms including pions and χ-fields. The Lagrangian, eq.(28), shows explicitly the density dependence of f_π^2, m_χ^2 and m:

$$
\begin{aligned}
f_\pi^{*2} &= \frac{f_\pi^2}{f_d^2}(f_d+\chi_0)^2 = f_\pi^2(1+\frac{\chi_0}{f_d})^2\\
m_\chi^{*2} &= \frac{\partial^2 V(\chi)}{\partial\chi^2}\mid_{\chi=f_d+\chi_0}=\frac{m_\chi^2}{f_d^2}(f_d+\chi_0)^2(1+3\ln\frac{f_d+\chi_0}{f_d})\\
&\simeq m_\chi^2(1+\frac{\chi_0}{f_d})^2(1+3\frac{\chi_0}{f_d})\simeq m_\chi^2(1+5\frac{\chi_0}{f_d})\\
m^* &= \frac{m}{f_d}(f_d+\chi_0)=m(1+\frac{\chi_0}{f_d}).
\end{aligned}
\tag{29}
$$

This is the result obtained in the previous section. Hence we can see that the mean field approximation in the chiral quark model coupled to dilaton is a good approximation in the large N_c limit with massive dilaton, $(\frac{m}{m_\chi})^2 << 1$, and gives

$$
\frac{m^*}{m}=\frac{f_\pi^*}{f_\pi}\cong\frac{m_\chi^*}{m_\chi}
\tag{30}
$$

as predicted by BR scaling. This is somewhat different from the observation of Nambu-Freund model [20] which consists of a matter field ψ and a dilaton field ϕ. After spontaneous symmetry breaking the matter field and

dilaton field acquire masses. There are universal dependences on the vacuum expectation values both for the matter field and the dilaton field. One can easily see that the universal dependence on the vacuum expectation value is no longer valid in our calculations due to the logarithmic potential from QCD trace anomaly.

In summary, we have found that the tadpole type corrections lead to the decreasing masses with increasing baryon density, while the radiative corrections induce Lorentz-symmetry-breaking terms. The pion remains massless at finite density in the chiral limit. In the context of large N_c approximation with large scalar mass, tadpoles dominate and the mean-field approximation is reliable, giving rise to a Lorentz-invariant Lagrangian with masses decreasing as the baryon density increases according to eq.(30). This analysis with large N_c approximation gives a clue to construct the Lorentz invariant lagrangian, *i.e*, dilated chiral quark model which incoporate the mended symmetry of Weinberg into chiral quark model, at finite density. The dilated chiral quark model can emerge in dense and hot medium – if it does at all – only through nonperturbative processes (e.g., large N_c expansion) starting from a chiral quark Lagrangian.

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Appendix I

We summarize the basic notation and definition of Thermo Field Dynamics(TFD) [21][22]. TFD is a real time operator formalism in quantum field theory at finite temperature. The main feature of the TFD is that thermal average of operator A is defined as the expectation value with respect to the temperature dependent vacuum, $|0(\beta)>$, which is introduced through Bogoliubov transformation.

$$\begin{aligned} < A > &\equiv Tr(Ae^{-\beta(H-\mu)})/Tr(e^{-\beta(H-\mu)}) \\ &= < 0(\beta) \mid A \mid 0(\beta) > \end{aligned} \tag{31}$$

where $\beta \equiv \frac{1}{kT}$, H is the total Halmiltonian of the system, and μ is the chemical potential. The propagator in TFD can be separated into two parts, i.e., the usual Feymann part G_F and density-dependent part G_D,

$$G_{\alpha\beta} = G_{F\alpha\beta} + G_{D\alpha\beta}. \tag{32}$$

For fermion with mass m,

$$G_{F\alpha\beta} = (\not{p} + m)_{\alpha\beta} \begin{pmatrix} \frac{1}{p^2-m^2+i\epsilon} & 0 \\ 0 & \frac{1}{p^2-m^2-i\epsilon} \end{pmatrix}$$

$$
G_{D\alpha\beta} = 2\pi i\delta(p^2 - m^2)(\not{p} + m)_{\alpha\beta} \begin{pmatrix} \sin^2\theta_{p_0} & \frac{1}{2}\sin 2\theta_{p_0} \\ \frac{1}{2}\sin 2\theta_{p_0} & -\sin^2\theta_{p_0} \end{pmatrix} \tag{33}
$$

with

$$
\begin{aligned}
\cos\theta_{p_0} &= \frac{\theta(p_0)}{(1+e^{-x})^{1/2}} + \frac{\theta(-p_0)}{(1+e^{x})^{1/2}}, \\
\sin\theta_{p_0} &= \frac{e^{-x/2}\theta(p_0)}{(1+e^{-x})^{1/2}} - \frac{e^{x/2}\theta(-p_0)}{(1+e^{x})^{1/2}},
\end{aligned} \tag{34}
$$

where α and β are Dirac indices and $x = \beta(p_0 - \mu)$.

The propagator of the scalar particle with mass m_s is given by

$$
\Delta(p) = \Delta_F(p) + \Delta_D(p), \tag{35}
$$

where

$$
\begin{aligned}
\Delta_F &= \begin{pmatrix} \frac{1}{p^2-m_s^2+i\epsilon} & 0 \\ 0 & \frac{-1}{p^2-m_s^2-i\epsilon} \end{pmatrix} \\
\Delta_D &= -2\pi i\delta(p^2 - m_s^2) \begin{pmatrix} \sinh^2\phi_{p_0} & \frac{1}{2}\sinh 2\phi_{p_0} \\ \frac{1}{2}\sinh 2\phi_{p_0} & \sinh^2\phi_{p_0} \end{pmatrix}
\end{aligned} \tag{36}
$$

with

$$
\begin{aligned}
\cosh\phi_{p_0} &= \frac{1}{(1-e^{-|y|})^{1/2}}, \\
\sinh\phi_{p_0} &= \frac{e^{-|y|/2}}{(1-e^{-|y|})^{1/2}},
\end{aligned} \tag{37}
$$

where $y = \beta p_0$.

Appendix II

In this Appendix II, we evaluate various intergrals in the text.

1. The contribution for the quark self energy from Fig. 3a(pion radiative corrections) is given by

$$\begin{aligned}
-i\Sigma_Q^{3a}(p) &= -(\frac{g_A}{f_\pi})^2\int\frac{d^4k}{(2\pi)^4}\frac{i}{(p-k)^2}(\not p-\not k)\gamma_5 T^a \\
&\quad\times(-2\pi)\delta(k^2-m^2)(\not k+m)(\not p-\not k)\gamma_5 T^a\sin^2\theta_{k_0} \\
&= i\frac{3}{4}(\frac{g_A}{f_\pi})^2\int\frac{d^4k}{(2\pi)^3}\frac{-(p-k)^2\not k+2(p-k)\cdot k(\not p-\not k)-m(p-k)^2}{(p-k)^2} \\
&\quad\times\delta(k^2-m^2)\sin^2\theta_{k_0} \\
&= i\frac{3}{4}(\frac{g_A}{f_\pi})^2\int\frac{d^4k}{(2\pi)^3}[-\not k-(\not p-\not k)-m]\delta(k^2-m^2)\sin^2\theta_{k_0} \\
&= -i\frac{3}{4}(\frac{g_A}{f_\pi})^2(\not p+m)\frac{1}{4\pi^2}\int dk_0\bar{k}\sin^2\theta_{k_0} \\
&= -i\frac{3}{8\pi^2}(\frac{g_A}{f_\pi})^2(\not p+m)I \qquad (38)
\end{aligned}$$

where $\bar{k}$ denotes $|\vec{k}|$. This gives eq.(11) in section 4.

2. We evaluate J^0 and $\vec{J}\cdot\vec{p}$ in eq.(15), which are needed in the evaluation of the quark mass correction from χ-field in Fig. 3b. Firstly let us consider J^0,

$$\begin{aligned}
J^0 &= \int\frac{d^4k}{(2\pi)^3}\frac{k_0}{2m^2-2p\cdot k-m_\chi^2}\delta(k^2-m^2)\sin^2\theta_{k_0} \\
&= \int\frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2}\frac{1}{2m^2-2p\cdot k-m_\chi^2}\sin^2\theta_{k_0}
\end{aligned}$$

$$= \frac{1}{8\pi^2}\int_0^{k_F} d\bar{k}\bar{k}^2 \frac{1}{2\bar{p}\bar{k}} \ln(\frac{2\bar{p}\bar{k} + 2m^2 - 2Ek_0 - m_\chi^2}{-2\bar{p}\bar{k} + 2m^2 - 2Ek_0 - m_\chi^2}) \tag{39}$$

Note that due to the factor $sin^2\theta_{k_0} \mid_{\beta\to\infty} \to \theta(\mu - k_0)$, k_0 is maximally order of chemical potential, i.e., $k_0 \sim \mu$. Assuming the large dilaton mass, $m_\chi^2 >> \mu^2 > m^2 (\sim E^2)$,

$$\begin{aligned} J^0 &\cong \int \frac{d^3\vec{k}}{(2\pi)^3}\frac{1}{2}\frac{1}{2m^2 - 2p\cdot k - m_\chi^2}\sin^2\theta_{k_0} \\ &= -\frac{1}{12\pi^2}\frac{1}{m_\chi^2}(\mu^2 - m^2)^{3/2} \end{aligned} \tag{40}$$

Similarly,

$$\vec{J}\cdot\vec{p} = \int \frac{d^4k}{(2\pi)^3}\frac{\vec{p}\cdot\vec{k}}{2m^2 - 2p\cdot k - m_\chi^2}\delta(k^2 - m^2)\sin^2\theta_{k_0} \tag{41}$$

can be approximated

$$\begin{aligned} \vec{J}\cdot\vec{p} &\cong \frac{1}{8\pi^2}\frac{4E^2}{m_\chi^4}\int dk_0 k_0^2 \bar{k} \\ &= \frac{1}{8\pi^2}\frac{E^2}{m_\chi^4}\theta(\mu - m)[\mu(\mu^2 - m^2)^{3/2} \\ &\quad + \frac{m^2\mu\sqrt{\mu^2 - m^2}}{2} - \frac{m^4}{2}\ln(\frac{\mu + \sqrt{\mu^2 - m^2}}{m})] \end{aligned} \tag{42}$$

3. The contribution from Fig. 5 for the dilaton self-energy, eq.(23), is given by

$$-i\Sigma_\chi^{(5)}(p^2) = -(\frac{m}{f_d})^2 tr\int \frac{d^4k}{(2\pi)^4}(\not{p} + \not{k} + m)(\not{k} + m)(-2\pi)$$

$$
\begin{aligned}
& \times \ [\frac{i}{(p+k)^2-m^2}\delta(k^2-m^2)\sin^2\theta_{k_0} \\
& + \frac{i}{k^2-m^2}\delta((p+k)^2-m^2)\sin^2\theta_{p_0+k_0}] \\
= \ & 8i(\frac{m}{f_d})^2\int\frac{d^4k}{(2\pi)^3}[\frac{p\cdot k+2m^2}{p^2+2p\cdot k}+\frac{-p\cdot k+2m^2}{p^2-2p\cdot k}]\delta(k^2-m^2)\sin^2\theta_{k_0} \\
= \ & 8i(\frac{m}{f_d})^2\int\frac{dk_0dx}{(2\pi)^3}\bar{k}d\bar{k}^2[\frac{Ek_0-\bar{p}\bar{k}x+2m^2}{p^2+2Ek_0-2\bar{p}\bar{k}x} \\
& + \frac{-Ek_0+\bar{p}\bar{k}x+2m^2}{p^2-2Ek_0+2\bar{p}\bar{k}x}]\delta(k^2-m^2)\sin^2\theta_{k_0} \\
= \ & -i(\frac{m}{f_d})^2\frac{1}{\pi^2}\int dk_0\bar{k}[2+\frac{p^2-4m^2}{4\bar{p}k}\ln(\frac{p^2+2Ek_0-2\bar{p}\bar{k}}{p^2+2Ek_0+2\bar{p}\bar{k}}) \\
& - \frac{p^2-4m^2}{4\bar{p}k}\ln(\frac{p^2-2Ek_0+2\bar{p}\bar{k}}{p^2-2Ek_0-2\bar{p}\bar{k}})]
\end{aligned} \tag{43}
$$

In the large dilaton mass limit, $(\frac{m}{m_\chi})^2 << 1$,

$$
\begin{aligned}
-i\Sigma_\chi^{(5)}(p^2) \ \cong \ & -i\frac{8}{\pi^2}(\frac{m}{f_d})^2\int dk_0\bar{k}[\frac{m^2}{p^2}-\frac{1}{3p^4}(\bar{p}^2\bar{k}^2+3E^2k_0^2)]\times\sin^2\theta_{k_0} \\
\cong \ & -i\frac{8}{\pi^2}(\frac{m}{f_d})^2\int dk_0\bar{k}[\frac{m^2}{p^2}-\frac{E^2k_0^2}{p^4}] \\
= \ & -i\frac{m^2}{m_\chi^2}\frac{8}{\pi^2}(\frac{m}{f_d})^2[\frac{1}{2}\theta(\mu-m)(\mu\sqrt{\mu^2-m^2}-m^2ln(\frac{\mu+\sqrt{\mu^2-m^2}}{m}) \\
& - \frac{E^2}{m_\chi^2}(\frac{\mu(\mu^2-m^2)^{3/2}}{4m^2}+\frac{\mu\sqrt{\mu^2-m^2}}{8} \\
& - \frac{m^2}{8}\ln(\frac{\mu+\sqrt{\mu^2-m^2}}{m})]
\end{aligned} \tag{44}
$$

where $\bar{p}$ and $\bar{k}$ denote $|\vec{p}|$ and $|\vec{k}|$ respectively, and $p^2 = m_\chi^2$ is used. It is suppressed by order of $\frac{m^2}{m_\chi^2}$ compared to the tadpole contribution.

Figure Captions

Fig.1 Self-energy diagrams for pions. Solid lines represent the constituent quarks and dashed lines are for pions. D(F) represents thermal(Feynman) propagator.

Fig.2 Tadpole type diagram for quark. Solid lines represent the constituent quarks and wavy lines are dilaton field respectively.

Fig.3 Self-energy diagram for quark.

Fig.4 Tadpole type diagram for dilaton field.

Fig.5 Self-energy diagram for dilaton field.